ALGEBRA – I[31111]

1. Prove Basis Representation Theorem
2. State and prove fundamental theorem of homomorphism.
3. State and prove Sylow's third theorem.
4. Show that a field is an integral domain.
5. Let R be a commutative ring. An ideal P of R is prime iff R/ P is an integral domain.
6. In a PID an element is prime if and only if it is irreducible.

ALGEBRA – II[31121]

1. Let V be a finite dimensional vector space and W, a subspace of V. Then show that dim A(W) = dim V – dim W.
2. State and prove Remainder Theorem.
3. State and prove the fundamental theorem of Galois Theory.
4. State and prove Sylvester' Law.
5. Show that the eigen values of a Hermitian matrix are real numbers.

ANALYSIS – II[31122]

1. State and prove Cauchy's General Principle of Convergence.
2. A series Σf_n (x) converges uniformly (and absolutely) on a set S if there exists a convergent series ΣM_n of non-negative terms M_n such that $|f_n \leq(x)| \leq M_n$ for all $x\in S$ and $n\in$ N.
3. State and prove Uniqueness Theorem for Power Series.
4. Describe the fundamental properties of measurable sets.
5. State and prove Fatou's lemma.

TOPOLOGY – II[31123]

1. State and prove Tychonoff's Theorem.
2. State and prove Cantor's intersection theorem.
3. Show that if Y is Hausdorff, then C(X, Y) is Hausdorff in the compact-open topology.
4. State and prove Ascoli's theorem.
5. Prove that every complete metric space is of the second category as a subset of itself.

PARTIAL DIFFERENTIAL EQUATIONS[31124]

1. Solve the differential equation $(x^2D^2 + xyDD' + 3y^2D')\, z = (x^2 – y)^{n/2}$. Find the radius of curvature at (a/2, a/2) on the curve $2ay^2 = (2a – x)\, 3$.
2. Find the surface which intersects the surfaces of the system u(x - 2y) = c (3u - 7) orthogonally and which passes through the circle $2x^2 - y^2 = 1$, u = 1. Here, c is a real parameter.
3. Find the Fourier sine transform of F(x) = x such that 0 < x < 2.

4. Explain Hankel transform of the derivatives of a function.
5. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ if the string of length 2a is originally plucked at the middle point by giving it an initial displacement d from the mean position.

DIFFERENTIAL GEOMETRY[31131]

1. State and prove Serret- Frenet formula.
2. Find the equation of the curvature and torsion of the involute of the curve c.
3. Obtain the surface equation of a cone with semi-vertical angle and find the singularities, the parametric curves, tangent plane at a point and a surface normal.
4. State and prove the differential equation of a geodesic.
5. State and prove Meusnier's theorem.

OPTIMIZATION TECHNIQUES[31132]

1. Reddy mikks produces both interior and exterior points from 2-raw materials $\boldsymbol{M1}$ and $\boldsymbol{M2}$. The following provides the basic data of the problem

   Tons of raw material per tin of maximum daily availability

	Ex.point	Int.point	(tons)
Raw material $\boldsymbol{M1}$	6	4	24
Raw material $\boldsymbol{M2}$	1	2	6
profit/ton(rs.1000)	5	4	10

   A market survey indicate that the daily demand for interior point cannot exceed that for exterior paint by more than one tons. Also the maximum daily demand for interior paint is 2-times. Ready mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.
2. Solve the following LP model by the upper bounding algorithm.
   max Z=$\boldsymbol{3x_1} + \boldsymbol{5y} + \boldsymbol{2x_3}$ subject to $\boldsymbol{x_1} + \boldsymbol{y} + \boldsymbol{2x_3} \le \boldsymbol{14}$; $\boldsymbol{2x_1} + \boldsymbol{4y} + \boldsymbol{3x_3} \le \boldsymbol{43\ 0}$; $0 \le \boldsymbol{x1} \le \boldsymbol{4}, \boldsymbol{7} \le \boldsymbol{y} \le \boldsymbol{10}, \boldsymbol{0} \le \boldsymbol{x3} \le \boldsymbol{3}$.
3. Solve the following game $\begin{bmatrix} 1 & 7 & 2 \\ 0 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$
4. Solve minimum $((x)) = x_1{}^2 + x_2{}^2 + x_3{}^2$ subject to the constraints, $g1(x) = x_1 + x_2 + 3x_3 - 2 = 0$ $g2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$ using Jacobian method.
5. Show how the following problem can be made separable. Max z= $x_1x_2+x_3+x_1x_3$ Subject to $x_1x_2+x_3+x_1x_3 \le 10$, $x_1, x_2, x_3 \ge 0$.

ANALYTIC NUMBER THEORY[31133]

1. Prove that if both and are multiplicative then is also multiplicative.
2. State and prove Generalized Mobius inversion formula.
3. State and prove Euler – Fermat theorem
4. State and prove fundamental theorem of arithmetic
5. State and prove product formula

STOCHASTIC PROCESS[31134]

1. State and prove general ergotic theorem
2. Prove that if the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $1/\lambda$, then the events E form a Poisson process with mean λt.
3. If $m \leq 1$, the probability of ultimate extinctions is 1. If $m > 1$, the probability of ultimate extinction is positive root less than unity of the equation $p(s)=s$.
4. State and prove Yaglom's theorem.
5. State and prove Pollaczek – Khinchine Formula.

ANALYSIS – I[31112]

1. Prove that C set of complex number is a field.
2. State and prove Heine Borel theorem
3. Every totally hounded subset of a metric space is bounded.
4. State and prove cauchy's root test.
5. State and prove weierstrass theorem.

ORDINARY DIFFERENTIAL EQUATIONS[31113]

1. Solve $(D^2+2\ +1)\ y= x\cos x$
2. Which of the following sets of functions are linearly independent: (i) sin x, cos x, sin 2x (ii) 1+x, 1+2x, x^2
3. Show that x = 0 and x = –1 are singular points of $x^2(x+1)y''+ (x^2-1)Y' + 2y = 0$, where the first is irregular and the other is regular.
4. Solve the equation of order zero $xy''+y'+ xy = 0$ in series.
5. Solve $(xy \sin xy + \cos xy)\ y\ dx + (xy \sin xy - \cos xy)\ x\ dy = 0$

TOPOLOGY – I[31114]

1. In a recent survey of 400 students in a school, 100 were listed as smokers and 150 as chewers of gum; 75 were listed as both smokers and gum chewers. Find out how many students are neither smokers nor gum chewers.
2. Let A, B, C be three sets. Then, A × (B – C) = (A × B) – (A × C)
3. State and prove Zorn's lemma.
4. State and prove Tychonov's theorem
5. Prove that the collection of open spheres in a set X with metric d is a base for a topology on X.

ALGEBRA –II[31121]

1. State and inequality and parallelogram law
2. State and prove the fundamental theorem of Galois theory
3. Prove that any projection E on a vector space V is diagonalizable
4. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ = (a-b)(b-c)(c-a)
5. Let F be a finite field. Then there exists an irreducible polynomial of any given degree n over t .

ANALYSIS – I[31112]

1. State and prove mean value theorem
2. State and prove Cauchy's root test.
3. State and prove Cantor intersection theorem.
4. State and prove Heine-Borel covering theorem.
5. State and prove Young's theorem.

TOPOLOGY – I[31114]

1. State and prove Pasting lemma.
2. Prove that the finite cartesian product of connected space is connected.
3. State and prove Uryshon's lemma.
4. State and Proe Tiesz extenstion theorem.
5. State and prove Uryshon's Metrization theorem.

OPTIMIZATION TECHNIQUES[31132]

1. Solve the minimum-span problem for the network given below. The numbers on the branches represents the costs of including the branches in the final network.
2. In the following LP, compute the entire simplex tableau associated with $X_B = (x_1, x_2, x_5)^T$

$$\text{Minimize } z = 2x_1 + x_2$$

$$\text{Subject to } 3x_1 + x_2 - x_3 = 3$$

$$4x_1 + 3x_2 - x_4 = 6$$

$$x_1 + 2x_2 + x_5 = 3$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

3. Use bounded variable technique and solved max $Z = x_1 + 3x_2 - 2x_3$
   subject to $x_2 - 2x_3 \leq 1;\ 2x_1 + x_2 + 2x_3 \leq 8;\ 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 3, 0 \leq x_3 \leq 3$
4. Solve each of the following game whose pay-off matrix is given below.

		Player B		
		B_1	B_2	B_3
	A_1	9	3	1
Player A	A_2	6	5	4
	A_3	2	4	3

5. Write the Kuhn- Tucker conditions for the following minimization problem
   Min $f(x) = x_1^2 + x_2^2 + x_3^2$,
   $g_1(x) = 2x_1 + x_2 \leq 5$
   $g_2(x) = x_1 + x_3 \leq 2$
   $g_3(x) = -x_1 \leq 1$
   $g_4(x) = -x_2 \leq -2$
   $g_5(x) = -x_3 \leq 0$

DIFFERENTIAL GEOMETRY[31131]

1. Obtain the equation of the circular helix r =(acosu, asinu, bu), $-\infty < u < \infty$ where a>0 referred to s as parameter and show that the length of one complete turn of the helix is $2\Pi c$ where $c = \sqrt{a^2 + b^2}$.
2. Show that the length of the common perpendicular 'd' of the tangent at two near points distance 's' about is approximately given by $d = \frac{\kappa\tau s^3}{12}$

3. Find the locus of the centre of the spherical curvature.
4. Find the equation of the osculating plane, osculating circle, osculating sphere at the point (1,2,3) on the curve x=2t+1, y=$3t^2 + 2$, z=$4t^3$+3.
5. Find the equation of the curvature and torsion of the involute of the curve c.

NUMNERICAL ANALYSIS

1. Given the following equations.

   i) $x^4 – x-10 =0$
   ii) $x- e^{-x} =0$
   Determine the initial approximations to find the smallest positive root. Use these to find the roots correct to three decimal places with the following methods.
   a) Regula-Falsi method b) secant method
   b) Newton-Raphson method
2. Using Bairstow's method obtain the quadractic factor of the following equations:
   i) $x^4 -3x^3 +20x^2 +44x +54 =0$, with (p,q) =(2,2)
   ii) $x^4 –x^3 +6x^2 +5x +10 =0$, with (p,q) =(1.14, 1.42)
3. Using the Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

4. Obtain the piecewise quadratic interpolating polynomials for the function f(x) defined by the data

x	-3	-2	-1	1	3	6	7
f(x)	369	222	171	165	207	990	1779

Hence, find an approximate value of f(-2.5) and f(6.5).

5. Discretize the following initial value problem
   a) y` = t+y, y(1) =0,
   b)y` =$-y^2$, y(1) =1, using i) Euler method, ii) backward Euler method and iii) mid-point method.
   Compute y(1.2), using h=0.1 in each case.